$$\left.\begin{array}{l} z_{n+4} = e^{-2i\frac{\pi}{3}}.z_n \\ z_{n+8} = e^{-2i\frac{\pi}{3}}.z_{n+4} \end{array}\right\rangle \Rightarrow z_{n+8} = e^{-4i\frac{\pi}{3}}.z_n$$

$$\left(\overrightarrow{OM_n}, \overrightarrow{OM_{n+4}}\right) \equiv Arg\left(\frac{z_{n+4}}{z_n}\right)(2\pi) \equiv Arg\left(e^{-2i\frac{\pi}{3}}\right)(2\pi) \equiv -\frac{2\pi}{3}(2\pi)$$

$$\left(\overrightarrow{OM_{n+4}}, \overrightarrow{OM_{n+8}}\right) \equiv Arg\left(\frac{z_{n+8}}{z_{n+4}}\right)(2\pi) \equiv Arg\left(e^{-2i\frac{\pi}{3}}\right)(2\pi) \equiv -\frac{2\pi}{3}(2\pi)$$

$$\left(\overrightarrow{OM_{n+8}}, \overrightarrow{OM_n}\right) \equiv Arg\left(\frac{z_n}{z_{n+8}}\right)(2\pi) \equiv Arg\left(e^{4i\frac{\pi}{3}}\right)(2\pi) \equiv -\frac{2\pi}{3}(2\pi)$$

Et comme M_n, M_{n+4} *et* M_{n+8} appartient à (ζ) alors le triangle $M_nM_{n+4}M_{n+8}$ est équilatéral.

EX 33 :

1) $z^4 = -1 \Leftrightarrow z^4 = e^{i\pi}$ Les solutions sont : $z_k = e^{i\left(\frac{\pi+2k\pi}{4}\right)}$ Avec $k \in \{0, 1, 2, 3\}$

$z_0 = e^{i\frac{\pi}{4}}$; $z_1 = e^{3i\frac{\pi}{4}}$; $z_2 = e^{5i\frac{\pi}{4}} = e^{-3i\frac{\pi}{4}}$; $z_3 = e^{7i\frac{\pi}{4}} = e^{-i\frac{\pi}{4}}$

2) $\theta \neq k\pi\ ; k \in \mathbb{Z}$

$$\frac{z+i}{z-i} = e^{i\theta} \Leftrightarrow z+i = ze^{i\theta} - ie^{i\theta} \Leftrightarrow \left(e^{i\theta}-1\right)z = i\left(e^{i\theta}+1\right) \Leftrightarrow z = \frac{\left(e^{i\theta}+1\right)i}{e^{i\theta}-1}$$

$$\left.\begin{array}{l} Or\ e^{i\theta}+1 = e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}} + e^{-i\frac{\theta}{2}}\right) = \left(2\cos\frac{\theta}{2}\right)e^{i\frac{\theta}{2}}\ Euler \\ et\ e^{i\theta}-1 = e^{i\frac{\theta}{2}}.\left(e^{i\frac{\theta}{2}} - e^{-i\frac{\theta}{2}}\right) = \left(2i\sin\frac{\theta}{2}\right)e^{i\frac{\theta}{2}} \end{array}\right\rangle \Rightarrow \frac{\left(e^{i\theta}+1\right)i}{e^{i\theta}-1} = \frac{1}{tg(\frac{\theta}{2})} \Bigg\rangle \Rightarrow \frac{z+i}{z-i} = e^{i\theta} \Leftrightarrow z = \frac{1}{tg(\frac{\theta}{2})}$$

3) i n'est pas une solution de (E)

$$(z+i)^4 = -(z-i)^4 \Leftrightarrow \left(\frac{z+i}{z-i}\right)^4 = -1 \Leftrightarrow \frac{z+i}{z-i} = e^{i\frac{\pi}{4}}\ ou\ \frac{z+i}{z-i} = e^{3i\frac{\pi}{4}}\ ou\ \frac{z+i}{z-i} = e^{-3i\frac{\pi}{4}}\ ou\ \frac{z+i}{z-i} = e^{-i\frac{\pi}{4}}$$

Les solutions sont :

$$z_1 = \frac{1}{tg\frac{\pi}{8}} \quad ; \quad z_2 = \frac{1}{tg\frac{3\pi}{8}} \quad ; \quad z_3 = -\frac{1}{tg\frac{3\pi}{8}} \quad ; \quad z_4 = -\frac{1}{tg\frac{\pi}{8}}$$

Qarray.tn - Agra Blech - Qarray.tn